ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 39, Number 3, 2009

PROPERTIES OF INFINITE HARMONIC FUNCTIONS ON GRUSHIN-TYPE SPACES

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ABSTRACT. In this paper, we examine potential-theoretic and geometric properties of viscosity infinite harmonic functions in Grushin-type spaces, which are sub-Riemannian spaces lacking a group structure. In particular, we prove such functions enjoy comparison with Grushin cones. As a consequence, the distance function is viscosity infinite superharmonic, but we show that it is not necessarily viscosity infinite subharmonic and give geometric conditions when it is.

1. Introduction. The goal of this paper is to examine viscosity infinite harmonic functions in Grushin-type spaces from both the potential-theoretic and the geometric viewpoints. Motivated by the author's result in [**5**] that C^1_{sub} absolute minimizers are viscosity infinite harmonic (see Sections 3 and 4 for relevant definitions) and its improvement by Wang [**15**], who relaxes the regularity, we wish to establish the potential-theoretic properties of viscosity infinite harmonic functions. In particular, we wish to prove the following main theorem:

Main theorem. *Given a domain Ω and a function u, the following are equivalent.*

(1) *u is an absolute minimizer.*

(2) *u is viscosity infinite harmonic.*

(3) *u is potential harmonic.*

(4) *u enjoys comparison with Grushin cones.*

In addition, the corresponding "one-sided" statements hold. Namely, the following are equivalent.

(I) *u is an absolute sub (super)-minimizer.*

2000 AMS *Mathematics subject classification.* Primary 35H20, 49L25, 53C17, Secondary 17B70, 31B05.
Keywords and phrases. Grushin-type spaces, viscosity solutions, infinite Laplacian.
Received by the editors on May 31, 2006, and in revised form on September 28, 2007.
DOI:10.1216/RMJ-2009-39-3-729