SOME SERIES OF HONEY-COMB SPACES

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ABSTRACT. We study the topology and geometry of some series of closed connected orientable 3-manifolds constructed as honey-comb spaces. These manifolds are quotients of certain polyhedral 3-cells by pairwise identification of their boundary faces. We determine geometric presentations of the fundamental group and study the split extension of it. Then we describe geometric structures, homeomorphism type and covering properties of our manifolds which are shown to be cyclic coverings of the 3-sphere branched over known links with two components. Finally, we answer open questions on certain manifolds, defined by Kim and Kostrikin, and give a complete classification of them.

1. The Seifert-Weber dodecahedron space. Following [**16, 22**], we start with a detailed discussion on a classical example of hyperbolic closed 3-manifold, i.e., the *Seifert-Weber dodecahedron space*. Recall from [**5**] that the Coxeter group $[p, q, r]$ is generated by four elements α, β, γ and δ subject to the following relations:

$$\alpha^2 = \beta^2 = \gamma^2 = \delta^2 = 1$$
$$(\alpha\beta)^p = (\beta\gamma)^q = (\gamma\delta)^r = 1$$
$$(\alpha\gamma)^2 = (\beta\delta)^2 = (\alpha\delta)^2 = 1.$$

The group $[3, 3, 5]$ has order 14400, and is also generated by the elements α_1, β, γ and δ, where $\alpha_1 = (\alpha(\beta\gamma\delta)^5)^2\alpha$. Since these elements satisfy the relations of $[5, 3, 5]$, there is a unique epimorphism $\varphi: [5, 3, 5] \to [3, 3, 5]$ such that $\varphi(\alpha) = \alpha_1$, $\varphi(\beta) = \beta$, $\varphi(\gamma) = \gamma$ and