

ABSOLUTELY  $p$ -SUMMABLE  
SEQUENCES IN BANACH SPACES  
AND RANGE OF VECTOR MEASURES

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**ABSTRACT.** We provide characterizations of Banach spaces  $X$  such that, for a given  $p \geq 1$ , each absolutely  $p$ -summable sequence in  $X$  is included inside the range of an  $X$ -valued measure. Demanding the vector measure to be of bounded variation results in the class of Banach spaces having  $(q)$ -Orlicz property which corresponds to the (classical) Orlicz property for  $q = 2$  (here  $q$  is conjugate to  $p$ ). A similar result where the vector measure (of bounded variation) is allowed to take its values in a super space of  $X$  is also proved. In the end, examples are provided to illustrate the usefulness of the results.

**1. Introduction.** The recognition of sequences in a Banach space  $X$  which are contained inside the range of a vector measure is an important theme in the theory of vector measures. In this connection, quite a good deal is known regarding members of an  $X$ -valued sequence space  $E(X)$  being included inside the range of a vector measure. In a series of papers [5, 6, 7, 10], Pineiro and his collaborators were able to achieve a complete classification of Banach spaces  $X$  for  $E(X)$  consisting of all null sequences with or without the assumption of bounded variation on the vector measure  $\mu$  in question. Similar results pertaining to  $E(X)$  consisting of weakly  $p$ -summable sequence have been treated in [8, 9]. However, these results do not cover the case involving vector measures of bounded variation taking values in a superspace of  $X$ , which was accomplished by the author in [12] for weakly  $p$ -summable sequences in  $X$ . The methods employed in that paper also make it possible to provide an alternative proof of an earlier result of Pineiro [9] to the effect that Hilbert spaces are the only Banach spaces  $X$  in which null sequences, equivalently the unit ball, can be ‘wrapped’ inside the range

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