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# DECOMPOSING WITT RINGS OF CHARACTERISTIC TWO

MURRAY MARSHALL

The Witt rings considered here are the abstract Witt rings in the sense of [**6**]. A major problem is the following: Is every finitely generated Witt ring necessarily of elementary type? We restrict our attention to Witt rings of characteristic 2. This simplifies matters considerably. Just as an example, the classification of Witt rings with a 1-sided rigid element is pretty complicated [**2**]. If the characteristic is 2, then 1-sided rigids are automatically 2-sided so the classification is comparatively easy [**1**].

The main result here is to give necessary and sufficient conditions for a Witt ring of characteristic 2 to be a product (in the category of Witt rings) of group rings (see Theorem 1) or group rings and dyadic local types (see Theorem 2). This has similarities with the problem tackled in [**4**]. However the motivation here is different: we try to generalize the characterization of a product of two group rings given in [**3**, Theorem 3.10]. Once this result is established, it is used to obtain a characterization of elementary Witt rings of characteristic 2 (see Theorems 7 and 8). It is not clear how to generalize any of this to the characteristic $\neq 2$ case.

An earlier version of this paper [**7**] was submitted for publication and then later withdrawn in favor of the present paper. The results presented here, although they still leave something to be desired, are a substantial improvement over the results in [**7**].

Terminology and notation are as in [**3**, **6**, **8**]. Throughout, $R$ denotes a Witt ring of characteristic 2, and $G$ denotes the distinguished subgroup of units of $R$. The associated quaternionic pairing is denoted by $q : G \times G \to Q$. For $a \in G$, $D\langle 1, a\rangle$ denotes the value group of the 1-fold Pfister form $\langle 1, a\rangle$, i.e., $D\langle 1, a\rangle = \{x \in G | q(x, a) = 0\}$. Of course, we are assuming $\operatorname{char}(R) = 2$, so $-a = a$ holds for all $a \in G$.

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