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IDEAL CLASS GROUPS OF EXPONENT TWO AND ONE-CLASS GENERA OF BINARY QUADRATIC LATTICES

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Let K/K_0 be a relative quadratic extension of algebraic number fields. K equipped with the relative norm mapping forms a binary quadratic space over K_0. If R and R_0 denote the rings of algebraic integers of K and K_0, respectively, then R can be considered as a binary quadratic R_0-lattice on the quadratic space K. In this note, we will consider the relationship between the following two statements:

(A) R has genus class number one as quadratic R_0-lattice;

(B) all squares in the ideal class group of K are trivial.

In the classical case that K_0 is the rational number field and K is an imaginary quadratic field, (A) and (B) are equivalent by the Principal Genus Theorem of Gauss and the well-known correspondence between equivalence classes of integral binary quadratic forms and equivalence classes of ideals of R. We will show here that, in general, neither statement (A) nor (B) implies the other, and that the distinction between the two statements arises primarily from the nontriviality of the ideal class group of the ground field K_0.

Chowla proved in 1934 [**1**] that there exist only finitely many imaginary quadratic fields for which either of the equivalent conditions (A) or (B) hold. This finiteness for the number of fields satisfying (A) has been generalized to broader classes of fields by Pfeuffer [**9**]. In fact, under the assumption of the validity of either the generalized Riemann hypothesis or the Artin conjecture, there exist only finitely many CM-fields K (that is, totally imaginary quadratic extensions K of a totally real subfield K_0) for which (A) holds. The unproven hypothesis can be removed if one restricts either to the class of such fields having bounded degree, or to those which can be reached from the rational number field by a tower of relatively normal extensions.

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