

CONTINUA WITH A DENSE SET OF END POINTS

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ABSTRACT. The structure of metric continua with a dense set of end points is investigated. It is shown that a continuum has a dense set of end points if and only if it is either indecomposable or the union of two proper indecomposable subcontinua with connected intersection, each having a dense set of its end points lying outside the component containing the intersection and such that the intersection is an end continuum in both subcontinua.

A continuum means a compact connected metric space. Throughout this paper X always denotes an arbitrary continuum, and $C(X)$ is the hyperspace of all nonempty subcontinua of X equipped with the Hausdorff distance denoted by dist (see [5; §42, II, p. 47] for the definition).

If $K \in C(X)$ and if for each $L, M \in C(X)$ with $K \subset L \cap M$ we have either $L \subset M$ or $M \subset L$, then K is called an end continuum in X . Note that X is an end continuum in itself. In particular, if $K = \{p\}$, then the point p is called an end point of X (see [3; p. 660, 661]). The set of all end points of X is denoted by $E(X)$. Observe that $K \in C(X)$ is an end continuum in X if and only if K is an end point of the decomposition space X/K of the monotone upper semi-continuous decomposition of X whose only nondegenerate element is K .

Note that if we restrict our considerations to proper subcontinua of a given continuum, then what we call "end continua" here are called "terminal continua" in [4; Definition 4, p.461] and "absolutely terminal continua" in [2; Definition 4.1, p.34]. The same concerns points.

PROPOSITION 1. *The set $E(X)$ is a G_δ -set*

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