ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 16, Number 1, Winter 1986

WREATH PRODUCTS INDEXED BY PARTIALLY ORDERED SETS

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1. Introduction. In his classic paper Wreath powers and characteristically simple groups [6], Philip Hall gave a generalized definition of the wreath product of an infinite number of permutation groups indexed by a totally ordered set. Until this paper, wreath products had usually been described as ascending unions of finite wreath products. Hall's generalized definition allows one to construct more elaborate wreath products and, in fact, he constructs various characteristically simple groups using his definition.

Hall's definition has itself been generalized by Holland [7] and later by Dixon and Fournelle [1, 2] (see also McCleary [10]). As with the standard wreath product $A \wr B$ where one speaks of unrestricted and restricted wreath products, Holland's wreath products are unrestricted while those of Dixon and Fournelle are restricted. Both, however, consider the situation in which the indexing set is partially ordered rather than totally ordered.

The purpose of this paper is to extend some of the main results of Hall's paper [6] to the generalized (restricted) wreath products of [1, 2]. In particular, we let

$$W = \underset{\lambda \in \Lambda}{\mathrm{Wr}}\, G_\lambda$$

be the wreath product of the permutation groups G_λ indexed by the partially ordered set Λ. We say that Λ is *upwardly directed* if for all λ, $\mu \in \Lambda$, there is a $\tau \in \Lambda$ such that $\lambda \leqq \tau$ and $\mu \leqq \tau$. Similarly, we way that Λ is downwardly directed if for all λ, $\mu \in \Lambda$, there is some $\tau \in \Lambda$ such that $\tau \leqq \lambda$ and $\tau \leqq \mu$. We have the following analogues of Theorems A and B of [6].

THEOREM 1. *If Λ is upwardly directed and has no maximal element and if each G_λ is non-trivial and transitive, then $W' = W''$ and any normal subgroup of W' is normal in W.*

THEOREM 2. *If Λ is downwardly directed and has no minimal element and*

Received by the eclitors on May 2, 1983 and in revised form on April 23, 1984