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STRONGLY EXTREME POINTS IN $L^p(\mu, X)$

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ABSTRACT. A natural characterization of strongly extreme points in the unit ball of $L^p(\mu, X)$, where $1 < p < \infty$, is given. This characterization is compared to similar known results concerning strongly exposed points and extreme points.

Sundaresan [8] and Johnson [6] considered the problem of characterizing extreme points in the unit ball of $L^p(\mu, X)$, where X is a Banach space, (S, Σ, μ) is a measure space and $1 < p < \infty$. It is easily shown that sufficient conditions for f to be such a point are that $\|f\|_p = 1$ and, for almost all s in the support of f, the element $f(s)/\|f(s)\|$ is an extreme point of the unit ball of X. In [8] it is shown that these conditions are also necessary in the case that X is a separable conjugate space, S is a locally compact Hausdorff space and μ is a regular Borel measure; in [6] the same is shown in the case that X is any separable Banach space, S is a complete separable metric space and μ is a Borel measure. However, Greim [4] has produced an example of a nonseparable X and a norm one f in $L^p(\lambda, X)$, where λ is Lebesgue measure on [0, 1], such that f is an extreme point of the unit ball of $L^p(\lambda, X)$ but, for all s in [0, 1], the element $f(s)/\|f(s)\|$ is not an extreme point of the unit ball of X.

Johnson [7] and Greim [5] considered the similar problem of characterizing strongly exposed points. In [7], a sufficient condition is given for g in $L^q(\mu, X^*)$, where $p^{-1} + q^{-1} = 1$, to strongly expose f of norm one in $L^p(\mu, X)$. In [5], the proof that this condition is sufficient is used to show, in the case that X is a smooth Banach space, that f in $L^p(\mu, X)$ is a strongly exposed point of the unit ball if $\|f\|_p = 1$ and, for almost all s in the support of f, the element $f(s)/\|f(s)\|$ is a strongly exposed point of the unit ball of X. It is also shown in [5] that these conditions are necessary in the case that X is a separable Banach space (μ arbitrary) and in the case that μ is a Radon measure on a locally compact Hausdorff space (X arbitrary).

In this paper, the problem of characterizing strongly extreme points in $L^p(\mu, X)$ is considered. It will be shown (Theorem 1) that f is a strongly extreme point of the unit ball of $L^p(\mu, X)$ if $\|f\|_p = 1$ and, for almost all

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