ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 15, Number 4, Fall 1985

p-VALENT CLASSES RELATED TO CONVEX FUNCTIONS OF COMPLEX ORDER

M. K. AOUF

ABSTRACT. Let $C(b, p)$ ($b \neq 0$ complex, $p \geqq 1$) denote the class of functions $f(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k$ analytic in $U = \{z: |z| < 1\}$ which satisfy, for $z = re^{i\theta} \in U$,

$$\operatorname{Re}\left\{p + \frac{1}{b}\left(1 + \frac{zf''(z)}{f'(z)} - p\right)\right\} > 0.$$

From $C(b, p)$, we can obtain many interesting known subclasses including the class of convex functions of complex order, the class of p-valent convex functions and the class of p-valent functions f for which zf' is λ-spirallike in U. In this paper we investigate certain properties of the above mentioned class.

1. Introduction. Let $A_p (p \geqq 1)$ denote the class of functions $f(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k$ which are analytic in $U = \{z: |z| < 1\}$. Let Ω denote the class of bounded analytic functions $\omega(z)$ in U, satisfying the conditions $\omega(o) = o$ and $|\omega(z)| \leqq |z|$, for $z \in U$. Also, let $P(p)$ (with p a positive integer) denote the class of functions with positive real parts that have the form $P(z) = p + \sum_{k=1}^{\infty} c_k z^k$, which are analytic in U and satisfy the conditions $P(o) = p$ and $\operatorname{Re}\{P(z)\} > o$ in U.

For $f \in A_p$, we say that f belongs to the class $C(b, p)$ ($b \neq 0$ complex, $p \geqq 1$) if

$$\operatorname{Re}\left\{p + \frac{1}{b}\left(1 + \frac{zf''(z)}{f'(z)} - p\right)\right\} > 0, \qquad z \in U. \tag{1.1}$$

It is noticed that, by giving specific values to b and p, we obtain the following important subclasses studied by various authors in earlier works:

(i) $C(1, 1) = C$ is the well known class of convex functions;

(ii) $C(b, 1) = C(b)$, is the class of univalent convex functions introduced by Wiatrowski **[11]** and investigated in **[8]** and **[9]**;

(iii) $C(1, p) = C(p)$, is the class of p-valent convex functions considered by Goodman **[3]**;

Received by the editors on November 18, 1983 and in revised form on February 21, 1984.