ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 15, Number 1, Winter 1985

SPECTRUM OF NONPOSITIVE CONTRACTIONS ON C(X).

ROBERT E. ATALLA

ABSTRACT. Known results in the spectral theory of Markov operators are shown to have analogues which are valid for general contractions. For instance we discuss the group structure of the unimodular eigenfunctions, and the representation of an irreducible operator as a rotation of a compact group, followed by a multiplication.

1. Introduction. Throughout, X will be a compact T^2 space and $C(X)$ the continuous scalar valued functions on X, where the scalar field may be either the real or the complex numbers. T will be a contraction on $C(X)$, i.e., a linear operator will $\|T\| \leqq 1$. T is called a Markov operator in case $T \geqq 0$ and $T1 = 1$. In areas such as ergodic theory and spectral theory, the theory of Markov operators is much more developed than that of general contractions. The reason is that positivity is a great convenience when measures come into play. However there exists a device which enables us to bring positivity into the picture even when T is nonpositive. Let $F(T^*) = \{m \text{ in } C(X)^*: T^*m = m\}$, let m be an extreme point of the unit ball $F_1(T^*)$, and let φ_m be the Radon-Nikodym derivative $dm/d|m|$. This was introduced in [3] for the special case where $T^2 = T$, and used in [1] to transfer results from the ergodic theory of Markov operators to general contractions. In this paper we make use of the functions φ_m to prove results in spectral theory already well known for Markov operators [4, 6, 7, 8]. For instance we show that the unimodular eigenfunctions form a group under an operation a little more complicated than pointwise multiplication, and that if T is irreducible and the unimodular eigenfunctions "strongly separate" X, then T is essentially a rotation of a compact group, followed by a multiplication.

It should be noted that in contrast to the Markov case it is possible that $F(T^*) = \{0\}$. On the other hand it is easy to manufacture nontrivial examples: let R be a Markov operator, ϕ a unimodular continuous func-

AMS (MOS) *subject classification* (1970): Primary 47A10, secondary 47A65.
Key words and phrases: Contraction, Markov operator, eigenfunction, spectrum, invariant measure.
Received by the editors on May 12, 1983 and in revised form on August 17, 1983.