

THE PEDERSEN IDEAL AND THE REPRESENTATION OF C^* -ALGEBRAS

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ABSTRACT. Let A be a C^* -algebra, Z the center of A , and K the Pedersen ideal of A . It is proved that if ZA is dense in A , then K is equal to $(K \cap Z)A$. It is known from the Dauns-Hofmann representation theory that given a C^* -algebra A , there exists a C^* -bundle such that A is isometrically $*$ -isomorphic to the ring of sections which vanish at infinity. This, together with the above characterization of the Pedersen ideal, is used to prove that if ZA is dense in A , then K is isometrically $*$ -isomorphic to the ring of sections with compact support. Under the same assumption it is observed that $M(A)$, the multiplier algebra of A , is isometrically $*$ -isomorphic to the ring of bounded sections and that $M(K)$, the multiplier algebra of K , is $*$ -isomorphic to the ring of all sections.

1. Introduction. Let A be a C^* -algebra. If A is commutative, then $A = C_\infty(X)$, the continuous, complex-valued functions which vanish at infinity on a locally compact, Hausdorff space X . The algebra A contains the ideal $C_K(X)$, the functions with compact support. The multiplier algebra of A , $M(A)$, is equal to $C_b(X)$, the bounded, continuous functions on X and the multiplier algebra of $C_K(X)$ is equal to $C(X)$, the space of all continuous functions on X . The purpose of this note is to develop a non-commutative analogue of these relationships in terms of sections in a C^* -bundle. This will be done by use of the Pedersen ideal.

In order to develop an integration theory for arbitrary C^* -algebras, G.K. Pedersen introduced in [11] an ideal which is generally accepted as the non-commutative analogue of $C_K(X)$. This ideal will be referred to as the Pedersen ideal. Extensive studies of the Pedersen ideal and its multiplier algebra have been made by Lazar and Taylor [8], [9], Pedersen and Petersen [13], Akemann, Pedersen, and Tomiyama [1].

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The notation in this note is approximately that of [3]. The letter A will