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ON THE S-EQUIVALENCE OF SOME GENERAL SETS OF MATRICES

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ABSTRACT. To help classify the set of square matrices over a ring R under the relation of S-equivalence there is defined a module A_V together with a pairing on its torsion submodule, which is referred to as the Seifert system of V. It is shown that if R is a field, or R is a PID and det $(tV - V')$ has content 1, then the Seifert system characterizes an S-equivalence class. Furthermore, over a field S-equivalence is reducible to the notion of congruence.

1. Introduction. Two square matrices over a ring R are called S-equivalent if one can be derived from the other by a sequence of the following operations (or their inverses);

(1.1) Congruences, i.e., replacing V by PVP', with P unimodular over R,

(1.2) Row and column enlargements, i.e., replacing V by,

$$\text{(i)} \begin{bmatrix} 0 & 0 & 0 \\ 1 & a & b \\ 0 & c & V \end{bmatrix} \quad \text{or} \quad \text{(ii)} \begin{bmatrix} 0 & 1 & 0 \\ 0 & a & b \\ 0 & c & V \end{bmatrix}$$

To help classify matrices under this relation, we define a module A_V over the ring $R[t, t^{-1}]$, together with a pairing on its torsion submodule, which will be an invariant of the S-equivalence class of V. We refer to this as the Seifert system for V.

The geometric aspects of the study of S-equivalence have principally been developed in the work of Levine [5, 6, 7]. If $K \subseteqq S^{2n+1}$ is an odd dimensional knot, then any Seifert surface for K determines an integral matrix, called a Seifert matrix. S-equivalence can in this case be interpreted as the matrix theoretic analogue of adding or subtracting handles to these surfaces. S-equivalence actually characterizes the so-called simple embeddings (see Kearton [3]). The module A_V then corresponds to the integral homology of the universal abelian cover of $S^{2n+1} - K$, whose pairing is defined geometrically in Blanchfield [1].

Seifert matrices for knots can algebraically be characterized by the condition $\det(V - eV') = \pm 1$, where e is either $+1$ or -1. These matrices have been classified algebraically by Trotter [10, 11]. The results of

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