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CONJUGATE TYPE BOUNDARY VALUE PROBLEMS FOR FUNCTIONAL-DIFFERENTIAL EQUATIONS

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Dedicated to Professor Lloyd K. Jackson
on the occasion of his sixtieth birthday.

1. Introduction and preliminaries. Two-point boundary value problems (BVP's) for delay differential equations have been studied extensively, beginning with the work of G. A. Kamenskiĭ, S. B. Norkin and others (see [5], [7]) which was motivated by variational problems and problems in oscillation theory. L. J. Grimm and K. Schmitt [4] and Ju. I. Kovač and L. I. Savčenko [6] employed solutions of various differential inequalities for the study of two-point problems with retarded argument. In this paper, we show how a bilateral iteration procedure can be developed to yield existence and inclusion theorems for multipoint boundary value problems of conjugate type for nonlinear functional-differential equations.

Let $n > 1, I = [a, b]$ be a real compact interval, let $a = x_1 < x_2 < \cdots < x_k = b$, let $p_1(x), p_2(x), \ldots, p_n(x)$ be continuous on I, and define the linear differential operator L by

$$Ly = y^{(n)} + p_1(x)y^{(n-1)} + \cdots + p_n(x)y. \tag{1.1}$$

A Ju. Levin (see Coppel [1]) has obtained the following result which will play a central role in our work.

THEOREM 1.1. *Let L and I be as above, and suppose that L is disconjugate on I. Then the Green's function $G(x, s)$ for the k-point conjugate type boundary value problem*

$$Ly = 0, \tag{1.2}$$

$$y^{(i)}(x_j) = 0, i = 0, \ldots, n_j - 1, j = 1, \ldots, k, \tag{1.3}$$

where $\sum_{j=1}^{k} n_j = n$, satisfies the inequality

$$G(x, s)(x - x_1)^{n_1}(x - x_2)^{n_2} \cdots (x-x_k)^{n_k} \geqq 0, x_1 < s < x_k. \tag{1.4}$$

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