

MULTIPARAMETERED NONHOMOGENEOUS NONLINEAR EQUATIONS

K. A. YERION

1. Introduction. Many problems in mathematical physics can be formulated as nonhomogeneous nonlinear equations involving one or more parameters. The purpose of this paper is to study the structure of the solution set of two classes of such operator equations where the nonlinear operator is compact.

The first class of operator equations is the single-parametered equation

$$(1) \quad u = \lambda F(\lambda, u) + z,$$

where λ is a real parameter, z is a fixed element of a Banach space, and the nonlinear operator F satisfies certain positivity conditions. For the special nonhomogeneous case with $z = 0$, but $F(\lambda, 0) \neq 0$, W. R. Derrick and H. J. Kuiper [2] have shown the existence of an unbounded continuum of positive solutions (λ, u) . The first result extends their theorem to equation (1) and to an analogous multiparametered equation. Similar results for the homogeneous equation have been obtained by M. A. Krasnosel'skii [4], P. H. Rabinowitz [6], R. E. L. Turner [8] and others, in the single-parametered case; and by J. C. Alexander and J. A. Yorke [1] in the multiparametered case.

The second class of operator equations is given by the equation $u + F(u) = w$, where w is a fixed member of a Banach space. Assuming that $\|F(u)\| \leq \alpha\|u\|$, $0 < \alpha < 1$, and that $I + F$ is locally one-to-one, C. Panchal has shown the existence of a solution [5]. Our second main result is a continuity theorem for this equation in the special case that $w = \sum_{i=1}^n \lambda_i z_i$, where λ_i is a real parameter and z_i is a fixed member of the Banach space for $1 \leq i \leq n$.

To illustrate possible applications of these results, we consider two problems in nonlinear elasticity. The first problem is the motion of an inelastic string with one endpoint free. Under the assumption that the string is acted on solely by forces of gravity and tension, this problem has been considered by I. I. Kolodner [3], using classical analysis, C. A. Stuart [6], using operator theory, and others. We consider the case that the string is also acted on by an external force and show the existence of