ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 9, Number 4, Fall 1979

ON GOLDIE THEOREMS FOR QUOTIENT RINGS

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1. **Introduction.** It is well known that a ring has a quotient ring if and only if it satisfies the Ore condition. The structure of the quotient ring of a ring with the Ore condition has been intensively investigated. In particular, A. W. Goldie ([4], [5], and [6]) proved that a ring is prime Goldie if and only if its quotient ring is Artinian simple, and that a ring is semiprime Goldie if and only if its quotient ring is Artinian semisimple. The purpose of the present paper is to generalize the above theorems of Goldie to a ring with an A-biregular or weakly A-biregular quotient ring, where an A-biregular ring R is a ring whose stalks of the Pierce sheaf [9] induced by R are Artinian simple rings, and a weakly A-biregular ring is a ring whose stalks of the Pierce sheaf induced by R are Artinian semisimple. Our theorem is the following: Let R be a ring with the identity 1 in which every non-zero-divisor of the stalks of the Pierce sheaf is lifted to a non-zero-divisor of R. If the stalks are prime Goldie (semiprime Goldie), then the quotient ring of R exists and is A-biregular (weakly A-biregular). A counterexample will be given to show that the converse does not always hold. However, when no extra central idempotents are added to the quotient ring from those of R, the theorem is reversible. Moreover, as pointed out by G. Bergman [1], the set of idempotents of the quotient ring of a commutative ring may be larger than the set of those of the ring. We shall examine this fact for a non-commutative ring in detail, and our results characterize a weakly A-regular quotient ring being A-biregular, equivalently, semiprime Goldie stalks being prime Goldie. Since prime Goldie rings and semiprime Goldie rings admit only a finite number of central idempotents, our study is a generalization of Goldie theorems for quotient rings to a class of rings with infinite central idempotents.

The author wishes to thank the referee for his valuable suggestions and comments and for bringing [2] to our attention.

2. **Preliminaries.** The ring R is called a (left) Goldie ring if R satisfies the ACC condition on (left) annihilators, and if R has no infinite direct sum of (left) ideals ([7], p. 62). The ring R with the left Ore condition is a ring such that for any a, s in R with s a non-zero-divisor, there exist b, t in R with t a non-zero-divisor such that $ta = bs$. It is well known that R satisfies the (left) Ore condition if and only if it has a

Received by the editors on April 4, 1977 and in revised form on October 12, 1977.