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DISPERSION FREE WEIGHTS, MAXIMAL REFINEMENT IDEALS, AND ATOMICITY IN CERTAIN GENERALIZED SAMPLE SPACES

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ABSTRACT. An *HD* space is a generalized sample space each of whose subspaces has, as its logic, an orthomodular poset. We study *HD* spaces which allow full sets of dispersion free weights: in a sense, these sample spaces can be said to admit "hidden variables". For such a space, we show that any proposition in the logic contains a minimal proposition, and that any maximal refinement ideal of operations is generated by a single operation. While the former ensures that the logic is atomic, the latter provides a generalized analog of the classical "grand canonical operation". In fact, we obtain a stronger result concerning atomicity, namely, that the logic of any *HD* space in which all maximal refinement ideals are principal is atomic.

1. **Introduction.** Whereas classical mechanics admits the theoretical existence of a "grand canonical operation" which in a sense refines all possible operations on a given system, the concept of a grand canonical operation as such is to a large extent meaningless in quantum mechanics, where the Heisenberg Commutation Relations preclude the possibility of determining a system with absolute accuracy. The study of operational statistics undertaken by D. J. Foulis and C. H. Randall (see [3], [7], [8], [9], [10]) seeks to provide a generalized version of classical probability and statistics, a formulation of particular value in its applicability to quantum mechanical systems. In his ground-breaking paper [6], G. W. Mackey studies the "logic" of quantum mechanics, that is, the set of "questions" (propositions), ordered by implication, together with a certain family of probability measures on this poset. The Foulis-Randall approach has the additional feature that an analog of this "logic", and probability measures thereon, are naturally induced by a generalized sample space and the accompanying generalized weight functions, thereby emphasizing the role of the physical situation.

In this paper, we will be concerned with a special kind of generalized sample space, called an *HD* space (rigorous definitions will be given presently). The logic (in the Foulis-Randall sense) of an *HD* space is a complete orthomodular lattice, (see [1] for lattice theoretical terminology) and thus a natural generalization of the classical Boolean lattice of propositions. Being the model Mackey adopts, the complete orthomodular lattice can reasonably be considered typical of quantum logics.

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