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COMPACT OPERATORS, WEAKLY COMPACT OPERATORS, AND SMOOTH POINTS

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S. Heinrih [9] recently announced the following result.

THEOREM. *If E and F are Banach spaces and $K(E, F)$ is the Banach space of all compact operators from E to F, then a compact operator $L : E \rightarrow F$ is a smooth point in $K(E, F)$ if and only if* (a) *there is a unique point (up to scalar multiples) $x_0^* \in S(F^*)$ so that $\|L^*x_0^*\| = \|L^*\|$ and* (b) *$L^*(x_0^*)$ is a smooth point in E^*.*

In Theorem 2 of this note, we use this theorem to characterize those continuous function spaces $C(H, E)$ whose duals contain smooth points, and we give an explicit representation of those compact linear operators $T : C(H, E) \rightarrow F$ which are smooth points in $K(C(H, E), F)$. The paper then concludes with a proposition which shows how a deep geometrical result of James [10]—together with a recent result of Diestel and Seifert [6]—can be used to easily obtain a characterization of weakly compact operators $T : C(H) \rightarrow F$.

The general setting is as follows. Each of E and F is a Banach space, H is a compact Hausdorff space, and $C(H, E)$ is the Banach space (sup norm) of all continuous E-valued functions defined on H. If E is the scalar field, we shorten the notation to $C(H)$. If $T : C(H, E) \rightarrow F$ is a continuous linear map ($=$ operator), then the Riesz Representation Theorem asserts that there is a unique finitely additive vector measure $m : \Sigma \rightarrow B(E, F^{**})$ on the Borel σ-algebra Σ of H with values in the space of operators from E to the bidual of F such that (i) m has finite semivariation [7, Chapter 1], (ii) $|m_z| \in \mathrm{rca}(\Sigma)$ ($=$ the Banach space of all regular countably additive real-valued measures on Σ) for each $z \in F^*$ (here $|m_z|$ is the total variation of the measure $m_z : \Sigma \rightarrow E^*$ defined by $m_z(A)(x) = z(m(A)(x))$), and (iii) $T(f) = \int_H f dm$, $f \in C(H, E)$ (the integral converges in the norm). The reader may consult Goodrich [8], Brooks and Lewis [3], and Batt [1] for a full discussion of this setting. In particular we note that (1) if m is the representing measure for T, then $m(A)x = T^{**}(\xi_A x)$, where ξ_A is the characteristic function of the Borel set A and $x \in E$, and (2) m is countably additive and takes its values in $B(E, F)$ if T is compact or weakly compact. Further, if T is a

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