ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 9, Number 3, Summer 1979

TWO COUNTEREXAMPLES FOR MEASURABLE RELATIONS

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In [2] various definitions of measurable relations are made and discussed. Some questions are posed there concerning the equivalence of the various definitions under suitable restrictions of the spaces. It is our purpose to resolve two of these questions by constructing counterexamples.

1. **Definitons and notations.** The pair $(T, \mathscr{A})$ will denote a measurable space and a σ-algebra. We say $(T, \mathscr{A})$ is a *(complete) measure space* if $(T, \mathscr{A}, \mu)$ is a (complete) finite measure space for some measure μ.

A *Polish* space is a separable metrizable space which is complete under some metric. A *Souslin* space is a metrizable continuous image of a Polish space.

A relation $F : T \to X$ is a subset of $T \times X$. In conformity with [2], for $F : T \to X$, a relation, we denote by F the corresponding function into the set of subsets of X, and when we want to emphasize the properties of F as a subset of $T \times X$, we will refer to its graph $Gr(F)$ rather than F. If domain $F = T$, we call F a *multifunction* from T to X. As usual, we use the notations

$$F(A) = \{x \in X : (t, x) \in \mathrm{Gr}(F) \text{ for some } t \in A\}$$

and

$$F^{-1}(B) = \{t \in T : F(t) \cap B \neq \phi\}.$$

All relations under considerations are multifunctions in the present paper.

Let $F : T \to X$ be a relation from a measurable space $(T, \mathscr{A})$ to X, a metric space whose Borel σ-algebra is denoted by $\mathscr{B}$. We define, as in [2], F to be *measurable, (weakly measurable, $\mathscr{B}$-measurable, $\mathscr{C}$-measurable)* if $F^{-1}(B)$ is measurable for each closed (respectively, open, Borel, compact) subset B of X. Finally, $\mathscr{A} \times \mathscr{B}$ will denote the σ-algebra generated by $\mathscr{A}$ and $\mathscr{B}$ on $T \times X$.

The following theorem is proved in [2].

THEOREM. *Let $(T, \mathscr{A})$ be a measurable space, X be a separable metric space, and $F : \mathrm{T} \to X$ be closed valued. Consider the following statements*:

Received by the editors on September 16, 1977.