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DECAY OF SOLUTIONS OF SYMMETRIC HYPERBOLIC SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS

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ABSTRACT. We consider systems of the form $u_t + \sum_{j=1}^n A_j u_{x_j} = 0$, where the A_j's are constant $k \times k$ (hermitian) symmetric matrices, and u is a column vector of k components. We use Fourier transform to prove that non-static solutions decay in time at every point x. As a consequence, it follows that the energy of any such solution decays locally. More generally, we show that if $B(t)$ is a set which does not increase "too" fast, the energy in $B(t)$ of any non-static solution also decays.

1. **Introduction.** We consider systems of the form

$$\frac{\partial u}{\partial t} + \sum_{j=1}^{n} A_j \frac{\partial u}{\partial x_j} = 0, \tag{1}$$

where the A_j's are constant $k \times k$ (hermitian) symmetric matrices, and u is a column vector of k components. These are functions of the independent variables $t \in \mathbf{R}$ and $x = (x_1, \cdots, x_n) \in \mathbf{R}^n$. Systems of this type are the general form of a large number of equations of mathematical physics, such as Maxwell's equation, the equations of transmission lines, acoustics, elasticity (see Appendix in [7]), and even the equations of magnetogasdynamics (see [1]).

It is customary to discuss the above systems under additional assumptions on the matrices A_j. One such assumption is that the roots $\lambda = \lambda(p)$ of the characteristic equation

$$P(\lambda, p) = \det\left(\lambda I - \sum_{j=1}^{n} p_j A_j\right) = 0 \tag{2}$$

are all different from zero for $p \neq 0$, that is, the operator $\sum_{j=1}^n A_j \partial/\partial x_j$ is elliptic ([4], p. 178); or a fixed number of them never vanish for $p \neq 0$ ([3]); or the assumption contained in the definition of uniformity propagative systems of Wilcox ([7]). In our treatment we impose no restrictions on the A_j's other than those stated in the previous paragraph. This is important because there are systems, such as those of magnetogasdynamics, which possess roots $\lambda(p)$ that vanish for certain $p \neq 0$, but not identically. It has been shown that if a characteristic root $\lambda(p)$ is not identically zero then the set of those p where $\lambda(p) = 0$ is of measure zero ([1]). Since the $\lambda(p)$, for $|p| = 1$, are speeds of propagation of

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