ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 9, Number 2, Spring 1979

A NOTE ON SPECIAL CLASSES OF p-VALENT FUNCTIONS

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ABSTRACT. Let $V_k^{\lambda}(p)$ $(k \geqq 2,\ |\lambda| < \pi/2,\ p \geqq 1)$ denote the class of functions f analytic in $\mathscr{U}: \{z/|z| < 1\}$ having $(p-1)$ critical points there and satisfying

$$\limsup_{r\to 1^-} \int_0^{2\pi} \left| \operatorname{Re}\left\{ e^{i\lambda}\left(1 + \frac{re^{i\theta}f''(re^{i\theta})}{f'(re^{i\theta})}\right)\right\}\right| d\theta \leqq kp\pi \cos\lambda.$$

From $V_k^{\lambda}(p)$, we can obtain many interesting known subclasses including the class of functions of bounded boundary rotation and the class of p-valent functions $f(z)$ for which $zf'(z)$ is λ-spiral-like. In the present paper, the results obtained for $f \in V_k^{\lambda}(p)$ include a domain of values for $(1 + (zf''(z)/f'(z))$, a distortion theorem for $\operatorname{Re} e^{i\lambda} \log[f'(z)/z^{p-1}]$, and the Hardy classes to which f' and f belong.

1. **Introduction.** Let A_q $(g \geqq 1)$ denote the class of functions $f(z) = z^q + \sum_{n=q+1}^{\infty} a_n z^n$ which are analytic in $\mathscr{U}: \{z/|z| < 1\}$. For $f \in A_q$, we say f belongs to the class $V_k^{\lambda}(p,\ q)$ $(k \geqq 2,\ |\lambda| < \pi/2,\ p \geqq q,\ p$ an integer) if there exists $\delta > 0$ such that

$$(1) \qquad \int_0^{2\pi} \operatorname{Re}\left\{1 + \frac{re^{i\theta}f''(re^{i\theta})}{f'(re^{i\theta})}\right\} d\theta = 2p\pi \quad (1-\delta < r < 1)$$

and

$$(2) \quad \limsup_{r\to 1^-} \int_0^{2\pi} \left| \operatorname{Re}\left\{ e^{i\lambda}\left(1 + \frac{re^{i\theta}f''(re^{i\theta})}{f'(re^{i\theta})}\right)\right\}\right| d\theta \leqq k\,p\,\pi \cos\lambda.$$

Condition (1) implies that f has $(p-1)$ critical points in $\mathscr{U}$. Further, $V_2^{\lambda}(p,\ q)$ is the class of p-valent functions f for which zf' is λ-spiral-like in $\mathscr{U}$.

The class $V_k^{\lambda}(p,\ q)$ was recently introduced by the author [11]. For special parametrizations, $V_k^{\lambda}(p,\ q)$ coincides with several interesting classes. For instance, from condition (2), $V_k^{0}(1,\ 1)$ is the class of functions of bounded boundary rotation introduced by Löwner [5] and Paatero [7], [8]. The class $V_k^{\lambda}(1,\ 1)$ was investigated by Moulis [6] and Silvia [10], while $V_k^{0}(p,\ q)$ was recently studied by Leach [3].

Received by the editors on December 9, 1976.

AMS(MOS) Subject Classification. Primary 30A36; Secondary 30A32.

Key words and phrases: p-valent, bounded boundary rotation, λ-spiral-like, radius of convexity, Hardy classes.