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TORSION THEORY FOR NOT NECESSARILY ASSOCIATIVE RINGS

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1. **Introduction.** The purpose of this note is to investigate torsion theories in the category of not necessarily associative rings. Torsion theories for abelian groups and abelian categories were studied in Dickson's papers [4] and [5] and it is found that for associative and alternative rings radical and semisimple classes (in the sense of Kurosh and Amitsur) correspond to torsion and torsionfree classes, respectively.

We will be considering subclasses of some universal class of not necessarily associative rings, where a class is *universal* if it is homomorphically closed and hereditary (closed under taking ideals). Calling a class co-radical when its properties are dual to those of a radical class, it is well-known that a semisimple class need not be a co-radical class or vice versa. However, starting from Dickson's definition of a torsion theory, we nevertheless obtain a complete duality between torsion and torsionfree classes. Torsion classes turn out to be particular radical classes and torsionfree classes are special kinds of semisimple and co-radical classes. In Section 2 torsion and torsionfree classes will be characterized. In Sections 3 and 4 classes and constructions related to torsion theories will be investigated and further characterizations of torsion theories will be obtained. For fundamental definitions and properties of radical and semisimple classes we refer to [8] and [16].

2. **Characterizations of torsion theories.** All rings considered will be members of some fixed universal class of not necessarily associative rings. It is assumed that every class $\mathbf{X}$ considered contains the ring 0 and is an abstract class (that is, if $A \in \mathbf{X}$ and $A \cong B$ then $B \in \mathbf{X}$). Also remark that whenever we give an example we are tacitly assuming that our universal class is such that it contains the example. As usual, define the following functions $\mathcal{U}$ and $\mathcal{S}$ acting on classes of rings by $\mathcal{U}\mathbf{X} = \{A \mid A \text{ has no nonzero homomorphic image in } \mathbf{X}\}$. $\mathcal{S}\mathbf{X} = \{A \mid \mathbf{A} \text{ has no nonzero ideal in } \mathbf{X}\}$. Further, let us associate for any class $\mathbf{X}$ and to any ring A the ideals

$$\mathbf{X}(A) = \sum_{\alpha} (I_\alpha \lhd A \mid I_\alpha \in \mathbf{X})$$

and

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