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ON THE UNIVERSAL COMPACTIFICATION OF A CONE

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ABSTRACT. Herein we determine the universal (Bohr) compactification of a wide class of semigroups with the discrete topology; this class includes the positive additive rationals, p-adic rationals, reals, as well as the interior of a closed proper cone in R^n. Using the notion of the greatest semilattice homomorphic image, we describe the structure of the universal compactification of a closed proper cone in R^n supplied with the discrete topology.

0. **Introduction.** The universal compactification of the topological semigroup S is a pair (U, u) where U is a compact semigroup, $u : S \rightarrow U$ is a continuous homomorphism of S onto a dense subsemigroup of U, and for any other continuous homomorphism $f: S \rightarrow T$ with T a compact semigroup there is a continuous homomorphism $\bar{f} : U \rightarrow T$ such that $\bar{f} \circ u = f$. The pair (U, u) is known to exist for any topological semigroup (c.f. [13] or [7]) and is unique with respect to the obvious notion of equivalence.

First a comment on terminology: Several authors, including this author, have referred to (U, u) as the Bohr compactification of S. In [18], the Bohr compactification of S is a pair (B, b) where B is a compact commutative semigroup in which the semicharacters (i.e., continuous homomorphisms into the semigroup of complex numbers z with $|z| \leqq 1$) separate points, $b : S \rightarrow B$ is a continuous homomorphism of S onto a dense subsemigroup of B, and for any semicharacter γ on S there is a semicharacter $\bar{\gamma}$ with $\bar{\gamma} \circ b = \gamma$. One sees immediately that this definition is much more consistent with the terminology for topological groups; in this sense, the Bohr and universal compactifications may differ (e.g., any non-degenerate compact connected semilattice). We shall henceforth use this terminology.

Our purpose in this work is to make a contribution to the determination of the universal compactification of a closed proper cone in R^n. § 1 sets forth definitions, notation, references, and some general information. In § 2 we develop some techniques for computing certain closed subgroups of the Bohr compactification of dense subgroups of R^n with the discrete topology; at the end of the Section we give examples using the techniques developed. In § 3, we give a description of the universal compactification of a wide class of subsemigroups

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