ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 8, Numbers 1 and 2,
Winter and Spring 1978

INTRODUCTION TO MATHEMATICAL TECHNIQUES FOR CONTINUOUS PROBLEMS

Certain special, and very important, nonlinear partial differential equations can be understood, and at times solved, through inverse spectral methods. The term "solve" refers to something more explicit than an existence proof, but—in most cases—less explicit than an integral representation such as is used in the classical solutions of the linear heat or wave equations. Many of the equations in question admit the multi-soliton solutions which we mentioned in the preface. These particular solutions can be expressed in terms of elementary functions or, under periodic boundary conditions, in terms of the Riemann theta function. The general initial value problem, however, can only be reduced to a sequence of linear problems for which closed-form solutions are not to be expected.

There are several types of mathematical problems which have been treated in the literature:

(1) To derive (preferably, from a unifying point of view) the special equations.

(2) To understand the auxiliary linear problems encountered in step 1. (This usually means: study a direct and inverse spectral problem of novel form.)

(3) To use the information from step 2 in a qualitative analysis of the solution of the nonlinear p.d.e., and to study the effects small perturbations have on the solution.

The papers in this section of the Proceedings only present a sample of the many novel ideas and techniques that have been developed in response to the problems just listed. In this introduction, we try to identify some of the major trends and open questions.

0. The Korteweg-deVries equation. The reader who is new to the subject will probably find it useful to keep a concrete example in mind while following the necessarily general outline we will present shortly. For this reason, we describe very briefly the basic features of the Korteweg-deVries equation, the first and still the most fundamental example of the inverse spectral methods.

0.1 The *KdV* equation. In suitable normalized form, the *KdV* equation is

$$q_t - 6qq_x + q_{xxx} = 0.$$