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ON CONTINUA OF PERIODIC SOLUTIONS FOR FUNCTIONAL DIFFERENTIAL EQUATIONS

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0. **Introduction.** Recently, certain arguments which originated in asymptotic fixed point theory have been used with remarkable success in the study of functional differential equations by G. S. Jones [**10**], S. N. Chow [**3**] and R. D. Nussbaum [**14, 15, 16**]. Our observations here are in this line and are directly motivated by results of Nussbaum in [**14**]. He considers differential-delay equations which can be transformed to the form $x'(t) = -\lambda f(x(t-1))$.

Our approach here will be to obtain an abstract existence result providing a continuum of non-trivial solutions for a nonlinear eigenvalue problem $F(x, \lambda) = x$ and apply it to obtain a continuum of non-trivial periodic solutions for certain differential-delay equations. Our abstract theorem is based on a kind of asymptotic version of Krasnosel'skiĭ's results in [**11, 12**] on expansions and compressions of a cone in a Banach space and this is due to G. Fournier and the author. The application then substantially relies on earlier work of E. M. Wright [**20**] and R. D. Nussbaum [**14**].

1. **Preliminaries.** We recall a few definitions and results which are essential for the abstract part of our considerations. We shall call a closed, convex subset P of a linear normed space a *cone* (with vertex 0) provided $x \in P$ implies $tx \in P$, $t \geqq 0$, and $x \in P$, $x \neq 0$, implies $-x \notin P$. If $r \in \mathbf{R}_+ = \{t \in \mathbf{R} \mid t > 0\}$ and X is either a cone or an infinite dimensional linear normed space then we fix the notation $B(r) = \{x \in X \mid \|x\| < r\}$, $S(r) = \{x \in X \mid \|x\| = r\}$. If X is a topological space and $A \subset X$, then cl A denotes the closure of A in X. A closed, connected subset of a topological space is called a continuum. Let X, Y be topological spaces and $f: X \to Y$ then $H_*(X)$ denotes the singular homology of X with coefficients in the field of rational numbers and $f_*: H_*(X) \to H_*(Y)$ denotes the induced homomorphism. In what follows an essential use will be made of the notion of the Lefschetz number in the generalized sense as given by J. Leray [**13**] and the fixed point index for metric ANR's developed by A. Granas in [**8**].

Let E be a graded vector space over the field of rational numbers, Φ an endomorphism of degree zero and $N(\Phi) = \bigcup_{n>0} \ker(\Phi^n)$. Then

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