## MULTI-PARAMETER SPECTRAL MEASURES, GENERALIZED RESOLVENTS, AND FUNCTIONS OF POSITIVE TYPE

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1. Introduction.

1.1. In this paper we extend the development of spectral triples as introduced in McKelvey [12] to the case of several parameters. Our central theme is the study of the interplay of certain classes of functions  $\{E(t), Q(\lambda), V(s)\}$  whose values are bounded operators on a complex Hilbert space. In the proto-type for the general situation these functions arise from a sequence,  $A^{j}$ ,  $j = 1, \dots, m + n$ , of m + n selfadjoint operators in the space H the last n of which are, in addition, positive. Corresponding to each operator  $A^{j}$  there is defined its resolution of the identity  $\mathbf{E}_{i,j}^{i} - \infty < t_{j} < \infty$ ,

$$\mathbf{A}^{j} = \int t^{j} d\mathbf{E}_{t^{j}}^{j},$$

its resolvent function  $\mathbf{Q}^{i}_{j}$ ,

$$\mathbf{Q}_{j}^{i} = \lambda^{j} (I - \lambda^{j} \mathbf{A}^{j})^{-1}, \text{ Im } \lambda^{j} \neq 0 \text{ unless } \lambda^{j} = 0,$$

and the unitary group  $U_{sj}^{j}$ ,

$$\mathbf{U}_{\mathbf{s}^{j}}^{j}=e^{-i\mathbf{s}^{j}\mathbf{A}^{j}},\quad -\infty < s^{j}<\infty$$

In case  $A^{j}$  is positive we prefer to work with the semi-group  $V_{s,i}^{j}$ ,

$$\mathbf{V}_{s^j}^j = e^{-s^j \mathbf{A}^j}, \quad s^j \ge 0.$$

Furthermore in this case the resolution of the identity vanishes on the half-axis  $t \leq 0$  and the resolvent is defined on the negative half-axis  $\lambda^{j} < 0$ .

We assume that the operators  $A^j$  commute pairwise, that is the resolutions of the identity  $E^j$  commute pairwise. Then all the operator families  $Q^j$ ,  $U^j$  and  $V^j$  commute pairwise and we may define the multiparameter operator functions E(t),  $Q(\lambda)$ , and V(s) according to the equations

$$\mathbf{E}(t^1, \cdots, t^{m+n}) = \prod_{j=1}^{m+n} \mathbf{E}_{t^j}^j, \ (t \in \Gamma_+)$$

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