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SINGULAR PERTURBATION OF SOME QUASILINEAR PARABOLIC EQUATIONS IN DIVERGENCE FORM

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Let $\Omega \subset E^n$, $n \geqq 2$, be a smooth domain. Consider for every $\epsilon > 0$ and $t > 0$ the initial value problem

$$\epsilon u_t - \sum_{i=1}^{n} \frac{d}{dx_i} a_i(x, t, u, u_x, \epsilon) + a(x, t, u, u_x, \epsilon)u + \epsilon f(x, t, \epsilon) = 0.$$

$$u(x, t, \epsilon) = 0, \quad \text{for } x \in \partial\Omega.$$

$$u(x, 0, \epsilon) = \mathring{u}(x, \epsilon), \text{ for } x \in \Omega.$$

$$(I_\epsilon) \quad \sum_{i=1}^{n} a_i(x, t, u, u_x, \epsilon) u_{x_i} \geqq \nu |\nabla u|^2.$$

$$|a_i(x, t, z, p, \epsilon)| \leqq M(|z| + |p|), \text{ and}$$

$$a(x, t, u, u_x, \epsilon) \geqq 0, \text{ for any } x \in \Omega, t > 0, \text{ and } z, p \in R \times R^n.$$

$$|f(x, t, \epsilon)| < M \text{ for any } x \in \Omega, t > 0, \text{ and } 0 < \epsilon < 1.$$

The purpose of this paper is to study the behavior of the solution of (I_ϵ) as $\epsilon \to 0$. The methods employed here are similar to [5], where the stability of some quasilinear parabolic equations in divergence form is considered.

Singular perturbation of quasilinear parabolic equations has been studied by Hoppensteadt [1]. He was able to obtain uniformly valid asymptotic expansions of the solution in terms of inner and outer expansions. However, many hypotheses were required to obtain his results. In particular, he required certain smallness criteria of the initial conditions. In this paper we are able to eliminate these special hypotheses. However, the equations we consider are of a more special type than was considered in [1] and our results are qualitative.

We will assume a unique classical solution exists for equations (I_ϵ).