

## THE NUMERICAL SOLUTION OF BOUNDARY VALUE PROBLEMS FOR STIFF DIFFERENTIAL EQUATIONS\*

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**ABSTRACT.** The numerical solution of boundary value problems for stiff ordinary differential equations is studied. Algorithms are developed which numerically construct asymptotic solutions of differential equations belonging to either a class of linear equations or quasi-linear second order equations. These algorithms avoid difficult stiff integrations by separately solving a reduced (or outer) problem away from boundary layers and adding appropriate solutions of boundary layer (or inner) problems where nonuniform convergence occurs. Numerical solutions generated in this manner are valid asymptotically; hence, they have the desirable feature of becoming more accurate as the equations become stiffer. Several numerical examples are presented which demonstrate the effectiveness of these methods for very stiff problems.

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