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LATTICE-VALUED BOREL MEASURES

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ABSTRACT. A Riesz representation type theorem is proved for measures on locally compact spaces, taking values in some ordered vector spaces.

In a series of papers ([4], [5], [6]), J. M. Maitland Wright has established, among other things, some Riesz representation type theorems for positive linear mappings from $C(X)$ to E, X being a compact Hausdorff space and E a complete (or σ-complete) vector-lattice. In this paper we prove these results (Theorem 4) by using the properties of order convergence in vector lattices.

We shall use the notations of ([2], [3]). For a compact Hausdorff space X, we denote by $C(X)$ the vector space of all continuous real-valued functions on X with sup norm, by $L(X)$ and $M(X)$ the dual and bidual of $C(X)$, respectively, and by $\beta(X)$ and $\beta_1(X)$ the sets of all bounded Borel and Baire measurable real-valued functions on X, respectively. In the natural order $C(X)$ is a vector lattice and $\beta(X)$ and $\beta_1(X)$ are boundedly σ-complete lattices. Also $L(X)$ and $M(X)$ are boundedly complete vector lattices and $C(X)$ is a sublattice of $M(X)$. Let $S(X)$ be the subspace of $M(X)$ generated by those elements of $M(X)$ which are suprema of bounded subsets of $C(X)$.

Let E be a vector lattice (always assumed to be over the field of real numbers). Order convergence, order closure (σ-closure), order continuity (σ-continuity) in vector lattices are taken in the usual sense ([1], [2], [3]). If A is a subset of E, let A_1 be the set of order limits, in E, of sequences in A, A_2 be the set of order limits of sequences in $A \cup A_1$ $(= A_1)$, and so on. Continuing this process transfinitely, if necessary, and taking the union of all these subsets, we get the order σ-closure of the set A. A vector subspace B of E we shall call monotone order closed (σ-closed), if for any net (sequence) $\{x_\alpha\}$, such that $x_\alpha \uparrow x$ in E, $x \in B$ ($x_\alpha \uparrow x$ means $\{x_\alpha\}$ is increasing and its sup is x). Now if A is a vector sublattice of a boundedly σ-complete vector lattice E, E_1 a monotone order σ-closed vector subspace of E, and $E_1 \supset A$, then $E_1 \supset A_1$ (A_1 as defined above); since A_1 is also a vector sublattice of E, $E_1 \supset A_2$, and so continuing this (transfinitely if necessary) we get $E_1 \supset$ order σ-closure of A. This result will be needed later. Monotone order continuity (σ-continuity) can be defined between ordered