ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 6, Number 2, Spring 1976

A TRANSFORMATION FORMULA FOR PRODUCTS ARISING IN PARTITION THEORY[1]

M. V. SUBBARAO AND V. V. SUBRAHMANYASASTRI[2]

ABSTRACT. We obtain a transformation formula involving Euler products. The formula can be utilized to obtain a large variety of partition-theoretic identities.

1. A transformation formula. Let $f(a, x)$ be the product given by

$$f(a, x) = \prod_{n=1}^{\infty} (1 - a^{\alpha(n)} x^n)^{g(n)/n}, \tag{1.1}$$

where $\alpha(n)$, $g(n)$ are totally multiplicative functions of n (that is, $\alpha(mn) = \alpha(m)\alpha(n)$, $g(mn) = g(m)g(n)$ for all positive integers m and n). Then we shall prove in this note that

$$\prod_{r=0}^{k-1} f(a, \omega^r x) = \prod_{d|k} \prod_{\delta|(k/d)} f(a^{(k/d)\alpha(d\delta)}, x^{k\delta})^{(g(d\delta)/\delta)\mu(\delta)}, \tag{1.2}$$

ω being a primitive k-th root of unity.

This result is a generalization of the identity proved earlier in [3]:

$$\prod_{r=0}^{k-1} \phi(\omega^r x) = \prod_{d|k} \{\phi(x^{kd})\}^{\sigma(k/d)\mu(d)}, \tag{1.3}$$

where

$$\phi(x) = \prod_{n=1}^{\infty} (1 - x^n), \tag{1.4}$$

and $\sigma(n)$ denotes the sum of the positive divisors of n. This is an important tool in deriving partition-theoretic identities such as the celebrated Ramanujan identity

Received by the editors on June 17, 1974, and in revised form on November 4, 1974.

[1]This research was supported in part by National Research Council Grant #3103.

[2]On leave from Sri Venkateswara University, India.

AMS (MOS) Subject classification(1970): 5A17, 5A19.

Key words and phrases: Euler and Jacobi identitites, primitive roots of unity.