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A TECHNIQUE IN PERTURBATION THEORY

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Introduction. A study of the effect of perturbations of differential equations depends on the method employed and on the nature of perturbations. One of the most used techniques is that of Lyapunov method and the other is the nonlinear variation of parameters formula [3]. These methods dictate that we measure the perturbations by means of a norm and thus destroy the ideal nature, if any, of the perturbing terms. Recently an effort was made to correct this unpleasant situation [1, 2].

In this paper, we wish to develop a new comparison theorem that connects the solutions of perturbed and unperturbed differential systems in a manner useful in the theory of perturbations. This comparison result blends, in a sense, the two approaches mentioned earlier and consequently provides a flexible mechanism to preserve the nature of perturbations. Our results will show that the usual comparison theorem in terms of Lyapunov function is imbedded as a special case in our present theorem and that the perturbation theory could be studied in a more fruitful way. An example is worked out to illustrate the results.

1. **A new comparison result.** We consider the two differential systems

$$y' = f(t, y), \; y(t_0) = x_0, \tag{1.1}$$

and

$$x' = F(t, x), \; x(t_0) = x_0, \tag{1.2}$$

where $f, F \in C[R^+ \times S(p), R^n]$. Here R^+ denotes the nonnegative real line, R^n the Euclidian n-space, $C[R^+ \times S(p), R^n]$ the class of continuous functions from $R^+ \times S(p)$ to R^n and $S(p) = [x \in R^n : \|x\| < p]$ where $\|\cdot\|$ is any convenient norm in R^n. Relative to the system (1.1), assume that

(H) the solutions $y(t, t_0, x_0)$ of (1.1) exist for all $t \geqq t_0$, are unique, continuous with respect to the initial data and $y(t, t_0, x_0)$ is locally Lipschitzian in x_0.

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