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SOME OSCILLATION CRITERIA FOR FOURTH ORDER DIFFERENTIAL EQUATIONS

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1. **Introduction.** In [2, Theorem 11.4, p. 374], W. Leighton and Z. Nehari showed that if q is a continuous function from $R^+ = [0, \infty)$ to $(0, \infty)$, and if

$$\int_0^\infty t^2 q(t)\, dt = \infty, \tag{1}$$

then every solution of

$$u'''' + qu = 0 \tag{2}$$

is oscillatory. (We call a continuous function f from R^+ to $R = (-\infty, \infty)$ *oscillatory* if and only if the set $\{t : t \text{ is in } R^+ \text{ and } f(t) = 0\}$ is unbounded. See the book of C. A. Swanson [4] for an excellent discussion of the work of Leighton and Nehari and many other authors). We shall give herein an oscillation criterion for

$$(p_3(p_2(p_1u')')')' + qu = 0 \tag{3}$$

which includes [2, Theorem 11.4]. In particular, with respect to

$$(ru'')'' + qu = 0, \tag{4}$$

our results generalize [2, Theorem 11.4] by showing that if

$$\int_0^\infty r(s)^{-1}\, ds = \infty \tag{5}$$

and

$$\int_0^\infty \left(\int_0^t (t-s)r(s)^{-1}\, ds \right) q(t)\, dt = \infty, \tag{6}$$

then every solution of (4) is oscillatory.

We shall also show that (6) can be weakened to

$$\int_0^\infty \left(\int_0^t (t-s)sr(s)^{-1}\, ds \right) q(t)\, dt = \infty, \tag{7}$$

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