

SOME UNSOLVED PROBLEMS CONCERNING COUNTABLY COMPACT SPACES

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Introduction. As is well known, the concept of countable compactness played an important role in much early work in general topology and was only ousted from its central position after A. Tychonoff showed that the more restrictive property of compactness was productive. Nevertheless present estimates of the gap between countable compactness and compactness cannot be very precise, for in each of the following questions if "countably compact" is replaced by "compact", then it is known that the resulting question has an affirmative answer. Yet the questions, as stated, appear to us to be quite challenging.

(1) Is every countably compact Hausdorff space that satisfies the first axiom of countability completely regular? (normal?)

(2) Is every countably compact quasi-topological group a topological group? (Raised by A. D. Wallace [34]).

(3) Does the existence of a countably compact Hausdorff space that is not compact imply the axiom of choice for sets of some given infinite cardinal?

(4) Is every countably compact Hausdorff space with a G_δ -diagonal metrizable? (Raised by B. A. Anderson in [1] and in an alternate form by R. W. Heath; Question 1 of [20].)

(5) Is every countably compact Hausdorff quasi-developable space a compact metric space? (Raised by H. R. Bennett in [5]).

(6) Is every countably compact perfectly normal space compact? (Raised by M. P. Berri, J. R. Porter and R. M. Stephenson; Problem 13 of [6]).

In this paper we are interested in the bearing that quasi-uniform spaces have upon the study of countably compact spaces that are not compact. We make no claim that the study of quasi-uniform spaces sheds light on all the above problems; our results are confined to partial solutions of problems 2, 4, 5 and 6. Nevertheless, it is known that a topological space is (countably compact, compact) if and only if its (upper semi-continuous, fine transitive) quasi-uniformity is pre-compact, and it is quite possible that the similarity of these charac-

Received by the editors August 20, 1973 and in revised form January 16, 1974.
AMS (1970) *subject classifications*. Primary 54-30, 54-52.

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