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COMPLETE ERGODICITY, WEAK MIXING AND STACKING METHODS

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Introduction. The purpose of this paper is to introduce stacking methods for constructing measure-preserving transformations, and to develop sufficient conditions which will insure that the resulting transformations are completely ergodic. It will be shown that while, in general, complete ergodicity does not imply weak mixing, in the case of these transformations it does.

1. **Preliminaries and Notation.** All transformations T considered in this paper will be defined on $[0, 1) = I$. They will be bimeasurable with respect to Lebesgue measure, invertible, and measure-preserving. Furthermore they will be constructed by a stacking method which fits into one of the four categories described in the next section. The following are a few preliminary definitions and theorems:

DEFINITION 1.1. T admits a k-stack if and only if there exists a set A with $\mu(A) > 0$ such that $T^kA = A$ and $\mu(A \cap T^iA) = 0$, $0 < i < k$.

THEOREM 1.1 (BLUM AND FRIEDMAN [1]). *T admits a k-stack if and only if the* kth *roots of unity are eigenvalues of T.*

In this paper, the term eigenvalue of T will be used in place of the eigenvalues of the induced operator U_T where $U_T(f)(x) = f(Tx)$.

LEMMA 1.1. *If T admits a k-stack then T^k is not ergodic.*

THEOREM 1.2. *If T^k is not ergodic, then for some prime $p \leqq k$, T admits a p-stack; furthermore, if k is prime, then T admits a k-stack.*

COROLLARY 1.2. *T^p is ergodic if and only if the* pth *roots of unity are not eigenvalues of T for prime p.*

2. **Stacking Methods.** Stacking methods were originally developed by Von Neumann and Kakutani to show the existence of ergodic transformations. R. V. Chacon generalized the method in [2].

Type 1a. The unit interval is divided into a stack consisting of h_1 subintervals of width w_1 each, and a residual R_1, and T is defined

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