ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 5 Number 1, Winter 1975

GENERALIZATIONS OF THE FIRST AXIOM OF COUNTABILITY

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ABSTRACT. This paper is a comprehensive survey of the concepts which generalize first countability, of the relations among the concepts, and of the major examples found in the literature on the topic.

0. **Introduction.** There are several reasons which one may give to indicate the value of generalizing the first axiom of countability. Among these are the following:

(1) To weaken assumptions in important theorems. For example, a closed image of a metric space is metrizable, if the range is assumed to be first countable.

(2) To study important properties. For example, if a real valued function f is continuous upon restriction to each compact subspace of a space X, then f is continuous on X, if X is a k-space.

(3) To study sequences and their properties. For example, in first countable spaces every accumulation point of a subset A is the limit of a sequence in A.

The emphasis in this article is on giving a comprehensive list of concepts which have been introduced to generalize first countability, from different points of view, along with examples and references. The reader may often judge the value of a concept by considering the number of references pertaining to it, as listed in our references in Section 1.

The structure of this survey is as follows: In Section 1, we present a list of definitions of concepts which generalize first countability. For standard terminology we follow Kelley [199], Nagata [290], and Thron [379], and the reader may note that there are some remarks concerning notation and terminology at the beginning of Section 1. The reader is advised to skim Section 1 and refer to it as needed as he would a dictionary. The history of the subject is briefly surveyed in Section 2, along with some motivation for the concepts and some

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AMS Subject Classification (1970): Primary 54D50, 54D55.

Much of the work on this paper was done while the writer was at St. John's University, Jamaica, New York.

Received by the Editors May 20, 1971 and, in revised form, January 31, 1974.