ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 4, Number 3, Summer 1974

# LIMIT THEOREMS FOR A CLASS OF MULTIPLICATIVE OPERATOR FUNCTIONALS OF BROWNIAN MOTION

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1. **Introduction.** Multiplicative operator functionals (MOF's) of Markov processes have proved to be an important extension of the concept of real valued multiplicative functionals as developed, for example, in Dynkin's treatise [2]. MOF's provide a unifying concept for the solution of many concrete problems encountered in transport theory, operations research, wave propagation in random media, and systems of partial differential equations. Pinsky's survey article [9] contains a fairly comprehensive account of the theory and applications of MOF's. The main results concerning MOF's consist of representation theorems and limit theorems. In this note we establish some limit theorems for certain MOF's of one and two dimensional Brownian motion; in addition, connections with differential equations are discussed. These results differ from previous limit theorems for MOF's in that the limiting operators associated with these MOF's do not define semigroups in contrast to similar results for MOF's of Markov chains and other Markov processes as proved in previous works. The author wishes to thank Reuben Hersh and Steve Rosencrans for helpful discussions during the preparation of this paper.

2. **Multiplicative Operator Functionals.** We use the notation of Dynkin [2] for Markov processes. Let $X = (x(t), \zeta, \mathcal{M}_t, P_x, x \in E)$ be a Markov process with state space $(E, \mathcal{B})$. Let $L$ be a fixed Banach space and let $\mathcal{L}$ be the space of all bounded linear operators on $L$. Following Pinsky [8] we say a *multiplicative operator functional* (MOF) of $(X, L)$ is a mapping $(t, \omega) \to M(t, \omega)$ of $[0, \infty] \times \Omega \to \mathcal{L}$ so that

(i) $\omega \to M(t, \omega)f$ is $\mathcal{M}_t$ — measurable for each $t \geqq 0, f \in L$.
(ii) $t \to M(t, \omega)f$ is right continuous a.s. for each $f \in L$.
(iii) $M(t + s, \omega)f = M(t, \omega)M(s, \theta_t\omega)f$ a.s. for each $t \geqq 0, f \in L$.
(iv) $M(0, \omega)f = f$ a.s. for each $f \in L$.

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[1]Research supported by NSF Grant GP-31811.
Received by the Editors June 22, 1973 and in revised form on June 29, 1973.
*AMS* 1970 *subject classifications.* 35K15, 47D05, 60J55.
*Key words and phrases.* Random evolutions, semigroups of operators, continuous additive functionals of Brownian motion, limit theorems, probabilistic solution of differential equations.