ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 4, Number 2, Spring 1974

AN APPLICATION OF LINEAR PROGRAMMING TO RATIONAL APPROXIMATION

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This is a report on a program for rational Chebyshev approximation of functions of either one or several variables. In addition, this program can be used to get good results for two types of simultaneous rational approximation. The algorithm used is a version of the differential-correction algorithm introduced by Cheney and Loeb [2].

Let $T \equiv \{t_1, \cdots, t_N\}$ be a finite set, where $T \subset R$ (the real line) or $T \subset R^k$. Let $f, \phi_1, \cdots, \phi_m, \Psi_1, \cdots, \Psi_n$ be functions defined on T. We then define the set of generalized rational functions

$$R_n^m \equiv \left\{ \frac{P}{Q} = \sum_{i=1}^{m} p_i\phi_i \Big/ \sum_{j=1}^{n} q_j\Psi_j \mid p_i, q_j \in R \text{ for all } i, j;\ Q > 0 \text{ on } T \right\}.$$

Our object is to choose $R \in R_n^m$ to minimize $\|f - R\|_T \equiv \max_{t \in T}|f(t) - R(t)|$.

The differential-correction algorithm for solving this problem is as follows:

(i) Choose any initial approximation $R_0 = P_0/Q_0 \in R_n^m$.

(ii) Having found $R_k = P_k/Q_k$, compute $\Delta_k \equiv \|f - R_k\|_T$, and choose P_{k+1} and Q_{k+1} as a solution of the following minimization problem (which can be solved by linear programming):

$$\text{minimize the expression} \quad \max_{t \in T} \frac{|f(t)Q(t) - P(t)| - \Delta_k Q(t)}{Q_k(t)}.$$

under the restrictions $|q_j| \leqq 1, j = 1, \cdots, n$.

Define $\Delta^* \equiv \inf_{R \in R_n^m} \|f - R\|_T$. We say that $R^* \in R_n^m$ is a best approximation if $\|f - R^*\|_T = \Delta^*$. Barrodale, Powell, and Roberts [1] have proved that if R_k is not a best approximation, then $Q_{k+1} > 0$ on T and $\Delta_{k+1} < \Delta_k$. Furthermore, $\Delta_k \to \Delta^*$, and this convergence is quadratic if $T \subset R$, $N \geqq m + n - 1$, $\phi_i = \Psi_i = t^{i-1}$ for all i, and a non-degenerate best approximation (i.e., one for which either numerator or denominator has greatest allowable degree) exists. The proof of quadratic convergence does not generalize easily to functions of

Supported in part by AFOSR 72-2271.
AMS (MOS) Subject classifications (1970) 41.17.