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A REMARK ON SIMPLE PATH FIELDS IN POLYHEDRA OF CHARACTERISTIC ZERO

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1. **Introduction.** A *path field* φ on a space X is a map $\varphi : X \to X^I$ such that $\varphi(x)(0) = x$, $x \in X$, and if $\varphi(x)(t) = x$ for $t > 0$ then $\varphi(x)$ is the constant path. φ is non-singular if $\varphi(x)$ is never the constant path. A non-singular path field φ is *simple* if $\varphi(x)$ is a simple arc for each x. Differentiable manifolds of (Euler) characteristic zero admit simple path fields while topological manifolds of characteristic zero are known to admit non-singular path fields [1]. The existence of simple path fields in the topological category is an open question. The purpose of this note is to observe that in the case of *triangulated* manifolds of characteristic zero it is easy to find a simple path field. In fact, every polyhedron K satisfying the so-called Wecken condition of characteristic zero admits a simple path field φ such that the track of $\varphi(x)$ is a broken line segment.

2. **Preliminaries.** Let K denote a finite polyhedron. We will not distinguish in the notation between K as a simplicial complex and K as the underlying space. If x is a point of K, then $\sigma(x)$ is the unique (open) simplex of K which contains x. Furthermore, if Δ represents the diagonal in $K \times K$, there is a special neighborhood of Δ given by

$$(1) \qquad \eta(\Delta) = \{(x, y) : \sigma(x) \text{ and } \sigma(y) \text{ have a common vertex}\}.$$

Each point $x \in K$ also has a special neighborhood defined by

$$(2) \qquad V(x) = \{y : (x, y) \in \eta(\Delta)\}.$$

Following R. F. Brown [2], we call a map $f : K \to K$ a *proximity map* if $f(x) \in V(x)$ for all $x \in K$.

If a, b are points of K in the same closed simplex, then $[a, b]$ will denote the segment from a to b. The following lemma is a somewhat stronger version of a lemma contained in [2] and [3]. The proof is the same except for additional observations.

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