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UNIQUENESS AND CONTINUOUS DEPENDENCE CRITERIA FOR THE NAVIER-STOKES EQUATIONS[1]

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I. **Introduction.** This paper is mainly of expository nature and deals with certain computational or practical questions regarding the Navier-Stokes equations. It is rather surprising that although the questions of existence and regularity have been pretty well answered by the important works of Leray [12] and others (see e.g. papers cited in the works of Finn [5], [6], Ladyzhenskaya [11], and Serrin [22]), usable criteria for stability and uniqueness were almost non-existent until very recently. These latter questions are in some ways much more difficult than those of existence and regularity, and, in fact, only the first rather crude steps in the determination of explicit uniqueness and stability criteria have been made (see e.g. Payne [17], [18]).

It would of course be very useful to have some a priori criterion involving the viscosity coefficient, the geometry of the domain, and the prescribed data, which would guarantee uniqueness of stationary solutions or stability of unsteady flows. It has long been known that if the viscosity coefficient is large enough, the domain is small enough and the data are small enough, then there do exist unique, stable solutions, but it was not known how small (large) was small (large) enough or just what the right measure of smallness (largeness) ought to be.

My primary interest in the Navier-Stokes equations has, therefore, been directed at the following questions:

1. The determination of explicit criteria for uniqueness in the steady state problem.
2. The determination of explicit criteria for convergence to steady state.
3. The determination of explicit growth criteria for solutions of the time dependent problem.
4. Solutions of the Navier-Stokes equations backward in time.

The discussion in this paper will be concerned primarily with these

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