ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 2, Number 2, Spring 1972

SPACES WITH COMPACT SUBTOPOLOGIES

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Introduction. In [1], Banach posed the problem of characterizing metric spaces which have a coarser compact metrizable topology. Banach asked if the space c_0 has the property. Klee [5] answered the question affirmatively. The purpose of this paper is to answer Banach's question in some special cases and to study a class of spaces containing all those with compact metrizable subtopologies. A γ space X is a topological space whose topology is finer than a compact Hausdorff topology.

§1 consists of a theorem which allows us to restrict our attention to Tychonoff spaces and several examples. In §2 we show that the class of γ spaces is closed under sums and products, but not under quotients. In §3 it is proved that an example of Sierpinski of a non-γ space admits a complete metric. Finally in §4 we prove a theorem which shows the abundance of non-γ spaces.

1. DEFINITIONS 1.1. A topological space X is a γ *space* if there is some compact Hausdorff space K and a continuous bijection from X onto K. A space X has *property* Γ if it is metrizable and its topology is finer than some compact metrizable topology. A topological space X is an *s space* if the family $C(X)$ of real continuous functions on X separates the points of X. A completely regular space X is a *Baire space* if the intersection of countably many dense open subsets of X is necessarily dense in X.

EXAMPLE 1.2. Every γ space is an s space. Hence every γ space is Hausdorff. However, the family $C(X)$ need not separate points and closed sets. That is, a γ space X need not be completely regular. Let $\{Z \mid |Z| \leqq 1\}$ be the closed unit disc in the plane. Let $\mathcal{U}$ be the usual topology for X and B the boundary of X in the plane. Topologize X as follows: A set U is open if

(1) $U \subset X \backslash B$ and $U \in \mathcal{U}$ or

(2) $U \cap (X\backslash B) \in \mathcal{U}$ and $x \notin \mathcal{U} - \mathrm{cl}(X\backslash(B \cup U))$ for $x \in U$. Thus, one sees that open sets contained in $X\backslash B$ are as usual and open sets about a point p of B consist of all points in some $\mathcal{U}$-open set U about p except for the points of $B\backslash\{p\}$ in U and unions of sets of this type. Call this topology τ. Now, (X, τ) is not completely regular. In

Received by the editors August 1, 1970 and, in revised form, December 1, 1970.

AMS 1970 *subject classifications.* Primary 54C10, 54A10; Secondary 54E50.