ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 1, Number 4, Fall 1971

EXAMPLES OF MODULAR ANNIHILATOR ALGEBRAS

BRUCE A. BARNES

1. **Introduction.** The theory of topological algebras with minimal ideals has developed to the point where now there is considerable literature on the subject. The earliest paper to appear is probably that of W. Ambrose [4] in which he introduced H^*-algebras, certain algebras that generalize the L^2 algebra of a compact topological group. Other early papers dealt with dual algebras (I. Kaplansky [26]), completely continuous algebras (I. Kaplansky [26]), and annihilator algebras (F. Bonsall and A. Goldie [18]). The most recent development has been the introduction of compact Banach algebras (K. Vala [50] and J. C. Alexander [1]). The definition of each of these algebras involves the topology of the algebra. In [54] B. Yood defined modular annihilator rings. We abbreviate modular annihilator to m.a. A ring A is a m.a. ring if every modular maximal left (right) ideal of A has a nonzero right (left) annihilator (note that this definition is purely algebraic). An equivalent formulation when A is semisimple is: A is a m.a. ring if and only if A/S_A is a radical ring, where S_A is the socle of A. There are many topological algebras with this property including (assuming semisimplicity) H^*-algebras, dual and annihilator Banach algebras, Banach algebras with dense socle, completely continuous normed algebras, and compact Banach algebras. The concept of m.a. algebras unifies the study of these various algebras. For example it is true for a general m.a. ring A that the structure space of primitive ideals of A is discrete in the hull-kernel topology; see [8, Theorem 4.2, p. 569]. This is proved for completely continuous Banach algebras in [27, Theorem 5.1], for annihilator B^*-algebras in [34, Corollary (4.10.15)] and for compact Banach algebras in [1, Theorem 6.1]. The basic properties of m.a. algebras can be found in [8] and [57].

The purpose of this paper is to provide examples of m.a. algebras which occur in analysis and in the theory of topological algebras with minimal ideals. Some of the results are taken from unpublished portions of [7]. A bibliography of papers concerning algebras with minimal ideals is included.

Received by the editors March 2, 1970 and, in revised form, June 22, 1970.
AMS 1970 *subject classifications.* Primary 46H05.