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QUANTUM-MECHANICAL SCATTERING THEORY FOR SHORT-RANGE AND COULOMB INTERACTIONS

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ABSTRACT. A rigorous account is given of time-dependent nonrelativistic quantum-mechanical scattering problems involving one or many particles. The discussion is carried out in the framework of Hilbert space. Results are given both for short-range and for Coulomb interactions.

I. The description of particles in quantum mechanics. A nonrelativistic spinless quantum-mechanical particle of mass m is described by assigning to each real number t, $-\infty < t < \infty$, an element ψ_t of $L^2(R^3)$ such that $\|\psi_t\| = 1$. ψ_t is called the *wave-function* or sometimes the *state* of the particle *at time t*. Introduce the Fourier transform $\tilde{\psi_t}$ of ψ_t by

$$\tilde{\psi_t}(\vec{k}) = \text{l.i.m.}\frac{1}{(2\pi)^{3/2}}\int_{R^3} e^{-i\vec{k}\cdot\vec{x}}\psi_t(\vec{x})d\vec{x} \ , \tag{1}$$

so that

$$\psi_t(\vec{x}) = \text{l.i.m.}\frac{1}{(2\pi)^{3/2}}\int_{R^3} e^{i\vec{k}\cdot\vec{x}}\tilde{\psi_t}(\vec{k})d\vec{k} \ . \tag{2}$$

Then the following partial interpretation of the wave-function can be given:

$|\psi_t(\vec{x})|^2$ is the position probability density (ppd) for the particle at time t,

$|\tilde{\psi}_t(\vec{k})|^2$ is the momentum probability density (mpd) for the particle at time t.

That is, if S is any (measurable) subset of R^3, then the probability that the particle is in S at time t is

$$P_p(S,t) = \int_S |\psi_t(\vec{x})|^2 d\vec{x} \ , \tag{3}$$

and the corresponding statement holds for momentum. Note that

$$P_p(R^3,t) = \int_{R^3} |\psi_t(\vec{x})|^2 d\vec{x} = \|\psi_t\|^2 = 1 \ , \tag{4}$$

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