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ON THE APPLICATION OF SEQUENTIAL AND FIXED-POINT METHODS TO FRACTIONAL DIFFERENTIAL EQUATIONS OF ARBITRARY ORDER

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ABSTRACT. This article analyzes the existence and approximation of solutions to initial value problems for nonlinear fractional differential equations of arbitrary order. Several new approaches are furnished in the environment of fractional differential equations, such as the sequential technique of Cauchy-Peano and the Leray-Schauder topological degree. In addition, some well-known ideas are optimized in the context of Banach's fixed-point theorem. A general version of Gronwall's inequality is also established. A recurring theme throughout the work is the incorporation of desirable qualities of the classical Mittag-Leffler function. A YouTube video presentation by the author designed to complement this work is available at http://tinyurl.com/Tisdell-JIEA.

1. Introduction. This article explores the existence and approximation of solutions to the following initial value problem (IVP) of arbitrary order $q > 0$

$$D^q \left(x - T_{\lceil q \rceil - 1}[x]\right)(t) = f(t, x(t)); \tag{1.1}$$

$$x(0) = A_0, \quad x'(0) = A_1, \dots , \ x^{(\lceil q \rceil - 1)}(0) = A_{\lceil q \rceil - 1}; \tag{1.2}$$

where $\lceil q \rceil$ is the integer such that $q - 1 < \lceil q \rceil \le q$; D^q represents the Riemann-Liouville fractional differentiation operator of arbitrary order $q > 0$ (a full definition is given in (2.2) a little later); $f : [0, a] \times D \subset \mathbf{R}^2 \to \mathbf{R}$; $T_{\lceil q \rceil - 1}[x]$ is the Maclaurin polynomial of order $\lceil q \rceil - 1$ of $x = x(t)$; $a > 0$ and the A_i are constants.

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