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FIXED POINT THEOREMS FOR CONVEX-POWER CONDENSING OPERATORS RELATIVE TO THE WEAK TOPOLOGY AND APPLICATIONS TO VOLTERRA INTEGRAL EQUATIONS

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ABSTRACT. In this paper we present new fixed point theorems for weakly sequentially continuous mappings which are convex-power condensing relative to a measure of weak noncompactness. Our fixed point results extend and improve several earlier works. As an application, we investigate the existence of weak solutions to a Volterra integral equation.

1. Introduction. During the last four decades several interesting studies relating to the existence of weak solutions to the Cauchy differential equation in Banach spaces have been presented. These studies were initiated by Szep [**23**] in 1971 and since then have been addressed by many investigators. We quote the contributions by Cramer, Lakshmikantham and Mitchell [**8**] in 1978 and more recently by Bugajewski [**4**], Cichon [**5, 6**], Cichon and Kubiaczyk [**7**], Mitchell and Smith [**17**], and O'Regan [**18–20**]. Motivated by the paper of Cichon [**6**], O'Regan [**18**] discussed in detail the problem (which was modeled off a first-order differential equation [**6**])

$$x(t) = x_0 + \int_0^t f(s, x(s))\, ds, \quad t \in [0, T]; \tag{1.1}$$

here $f\colon [0,T] \times E \to E$ and $x_0 \in E$ with E a real reflexive Banach space. The integral in (1.1) is understood to be the Pettis integral. Our main objective here is to establish existence results for the Volterra integral equation (1.1) in the case where E is nonreflexive. Our approach relies upon the concept of convex-power condensing operators with respect

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