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REPRESENTATION THEORY OF TOPOLOGICAL SELECTIONS OF MULTIVALUED LINEAR MAPPINGS WITH APPLICATIONS TO INTEGRAL AND DIFFERENTIAL OPERATORS

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1. Introduction. This paper is concerned with the characterization and representation theory of topological selections of closed linear relations in Banach spaces, and with some applications to differential and integral operators.

We first introduce some definitions. Let X and Y be real or complex Banach spaces and let M be a subspace (i.e., a vector subspace) of $X \times Y$. We can also view M as the graph of a multivalued linear mapping, and call it a linear relation in $X \times Y$. We say that R is an *algebraic selection* (or *algebraic operator part*) of M if R is the graph of a single-valued linear operator on $\operatorname{Dom} M$ into $\operatorname{Range} M$ such that $R \subset M$. Equivalently, $R = \operatorname{Null}\mathbf{P} := \{a \in M : \mathbf{P}(a) = 0\}$ for some algebraic projector $\mathbf{P}$ on M with $\operatorname{Range}\mathbf{P} = \{0\} \times M(0)$. If $\mathbf{P}$ is continuous, then R is called a *topological selection* (or *topological operator part*) of M. If, in addition, $\mathbf{P}(x, y) = \mathbf{P}(z, y)$ for all $(x, y), (z, y)$ in M, then R is called a *principal* topological selection of M. Let M^+ be a subspace of $Y^\# \times X^\#$, where $X^\#$ is the dual of X. An algebraic selection of M^+ is called a $w^\star$-topological selection of M^+ if the corresponding projector is $w^\star$-continuous.

We now summarize briefly the contents of this paper. In §2 we consider a general closed subspace (linear relation) M of $X \times Y$ and a $w^\star$-closed subspace M^+ of $Y^\# \times X^\#$ such that $M(0)$ and $M^+(0)$ are both finite dimensional. In Theorem 2.1 and Corollary 2.2, any topological selection is expressed by an adjoint subspace, while in Theorem 2.3 and Corollary 2.4, any $w^\star$-topological selection is expressed by a preadjoint subspace. These theorems and corollaries generalize and complete the corresponding theorems of Coddington and Dijksma [**1**]. §3 is concerned with the problem of characterizing a topological selection for a subspace of a linear relation in terms of a known topological selection of that relation, and with some related consequences. More specifically, suppose that M_1 is a known closed