

ON SOME RELATIONS BETWEEN THE EULER CLASS GROUP OF REAL VARIETIES AND TOPOLOGY

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ABSTRACT. The Euler class group of a smooth affine n -dimensional variety is in a certain sense the analogue of the “ n th cohomology group” of the variety. In this paper, we study the Euler class groups of smooth real affine varieties and the connections that these groups have with topology.

1. Introduction. Let A be a Noetherian ring and $J \subset A$ be an ideal such that J/J^2 is generated by n elements. Then, it is known (see for example, [1, Lemma 3.2]) that J can be generated by $n + 1$ elements. In general, however, J need not be generated by n elements.

For example, if m is the maximal ideal of the coordinate ring of the real circle, corresponding to a real point, then m/m^2 is generated by one element but m is not generated by one element, for the graph of any function which intersects the circle transversally at one point must cross the circle elsewhere.

One therefore poses the following general problem. Let A be a Noetherian ring. Let $J \subset A$ be an ideal such that J/J^2 is generated by n elements. When is J generated by n elements?

In view of the example of the coordinate ring of the circle, the special case of the general problem where the height of the ideal J is equal to the dimension of A is of interest and one poses (again) the following:

Question 1. Let A be a Noetherian ring of dimension n . Let $J \subset A$ be an ideal of height n . Suppose J/J^2 is generated by n elements. What is the obstruction to J being generated by n elements?

We briefly outline the answer to this question:

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