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ON THE DISCRETE COUNTERPARTS OF ALGEBRAS WITH STRAIGHTENING LAWS

MITSUHIRO MIYAZAKI

ABSTRACT. We study properties of a poset generating a Cohen-Macaulay algebra with straightening law. We show that if a poset P generates a Cohen-Macaulay algebra with straightening law, then P is pure and, if P is moreover Buchsbaum, then P is Cohen-Macaulay.

1. Introduction. DeConcini, Eisenbud and Procesi defined the notion of Hodge algebra in their article [**4**] and proved many properties of Hodge algebras. They also showed that many algebras appearing in algebraic geometry and commutative ring theory have structures of Hodge algebras. In fact, the theory of Hodge algebras is an abstraction of combinatorial arguments that are used to study those rings.

A Hodge algebra is an algebra with relations which satisfy certain laws regulated by combinatorial data. There exist many Hodge algebras supported on the same combinatorial data; however, there is one which is, in some sense, the simplest Hodge algebra with given combinatorial data, called the *discrete Hodge algebra*. For a given Hodge algebra, we call the discrete Hodge algebra with the same combinatorial data the discrete counterpart of it. DeConcini, Eisenbud and Procesi proved that:

• A Hodge algebra and its discrete counterpart have the same dimension.

• The depth of the discrete counterpart is not greater than the depth of the original Hodge algebra.

It is known that there is a Hodge algebra whose discrete counterpart has strictly smaller depth than the original one [**5**].

But if we restrict our attention to ordinal Hodge algebras (algebras with straightening laws, ASL for short), the influence of the combinatorial data on the ring theoretical properties becomes greater. So there

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