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THE ELIAHOU-KERVAIRE RESOLUTION IS CELLULAR

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ABSTRACT. We construct a regular cell complex which supports the Eliahou-Kervaire resolution of a stable ideal.

1. Introduction. A central object in the study of a homogeneous ideal $I \subseteq S = k[x_1, \dots, x_n]$ is the minimal free resolution, which encodes much information about the homological and combinatorial structure of the ideal. While algorithms to compute minimal free resolutions are known, the problem of describing them explicitly has proven intractable, even for monomial ideals. Thus, there has been a lot of work in recent decades describing the minimal free resolutions of well-behaved classes of monomial ideals.

One of the most important results in this vein is the Eliahou-Kervaire resolution [**12**], which elegantly describes the minimal resolution of a *stable* ideal in terms of its monomial generators. The stable ideals are a large class of monomial ideals containing (in characteristic zero) the *Borel-fixed* ideals. These occur as generic initial ideals of arbitrary ideals [**3, 13**], and so arise in many contexts.

Another approach has been to study non-minimal free resolutions. These reveal slightly less information than do minimal free resolutions, but are often much easier to describe. For example, the Taylor resolution [**21**] is a very clean (but usually highly non-minimal) resolution for any monomial ideal.

One of the most exciting recent developments in the study of resolutions has been the idea of *simplicial resolutions* [**2**], resolutions which can be described completely in terms of a simplicial complex. The Taylor resolution is simplicial, as are the minimal resolutions of "generic" monomial ideals. This idea was extended by Bayer and Sturmfels [**7**] to regular cell complexes, and later by Jöllenbeck and Welker [**15**] to

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