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LOCAL CONNECTIVITY OF LIMIT SETS

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ABSTRACT. This paper examines the dynamics of a continuous flow on a compact surface of genus greater than one with an orbit whose ω-limit set is locally connected. We show that if the orbit's lift to the Poincare disk limits to a rational point, then its ω-limit set contains a simple closed invariant curve that is not null homotopic. We also find sufficient conditions for the orbit's lift to stay a bounded distance from a geodesic with the same limiting point.

1. Introduction. When the ω-limit set is locally connected, the dynamics of a continuous flow on a compact surface M is linked to the existence of invariant simple closed curves on M. The focus of this paper will be when M is orientable and has genus $g > 1$. Theorems 5.1 and 5.2 state that such a curve exists if there is a positive orbit on M satisfying the following conditions: (a) its lift to the Poincare disk limits to a rational point and (b) either its ω-limit set is locally connected or the set of fixed points in its ω-limit set is totally disconnected. These theorems were proved by Markley for the torus in [**5**].

Note that if condition (a) fails, then $\omega(x)$ might be a Denjoy minimal set. If condition (b) fails, then $\omega(x)$ might look like the topologist's sine curve. In both cases the results no longer hold.

Markley also showed in [**5**] that if a positive orbit of a continuous flow on the torus has a lift $\mathcal{O}^+(\tilde{x})$ to the plane that goes to infinity, i.e., $|\tilde{x}t| \to \infty$ as $t \to \infty$, and its ω-limit set contains a moving point, then $\mathcal{O}^+(\tilde{x})$ will lie between two parallel lines. This result only holds for the torus. In [**6**] Markley and the author gave an example of a continuous flow on a compact surface of genus 2 with a positive orbit whose ω-limit set contains a nonperiodic orbit along with a simple closed curve of fixed points. The orbit does not wrap down on this simple closed curve in the usual way, and its lift to the Poincaré disk does not stay a bounded distance from a hyperbolic ray with the same limiting point on the unit circle. Theorem 6.5 shows that it was no accident that the

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