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ZETA FUNCTIONS AND REGULARIZED DETERMINANTS ON PROJECTIVE SPACES

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ABSTRACT. A Hermite type formula is introduced and used to study the zeta function over the real and complex n-projective space. This approach allows to compute the residua at the poles and the value at the origin as well as the value of the derivative at the origin that gives the regularized determinant of the associated Laplacian operator.

1. Introduction. Zeta functions on the sphere (and in general on a closed Riemannian manifold) were first introduced by Minakshisundaram and Peijel as extensions of the classical Riemannian zeta function [**12**]. An analytic definition, by a Mellin transform of the trace of the heat operator associated to the Laplacian in the standard metric, shows how the residua at the poles are given by the coefficients in the asymptotic expansion of the trace of the heat operator [**1**]. Such an expansion can be obtained in a large number of cases using global analysis [**16, 5, 3, 4**], and in particular the first coefficients in the case of the Laplacian on a Riemannian manifold can be computed using local invariants associated to the curvature tensor [**10**]. The constant term that corresponds to the value of the zeta function at the origin is important in physics [**15**] and in conformal theory [**2**], being associated to the conformal anomaly. On the other hand, the derivative of the zeta function at the origin gives the Atiyah regularized determinant of the Laplacian [**13**]. Early approaches to calculate these quantities give explicit results for the two-sphere [**15, 9**], while more recently an explicit formula for the residua has been obtained in [**7**] for the n-sphere, using a result that allows to write a Dirichlet series as a sum of classical Hurwitz zeta functions [**6**]. This method fails to compute the derivative, but an alternative one is provided by [**8**], where a factorization theorem for zeta regularized products is introduced and explicit results for the low-dimensional spheres are given.

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