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ON ABSOLUTE SUMMABILITY FACTORS

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ABSTRACT. The purpose of this paper is to determine the conditions for which $\sum a_n \lambda_\nu$ is summable $|T|_s$ whenever $\sum a_n$ is summable $|\overline{N}, p_n|_k$ where T is a lower triangular matrix with positive entries and row sums one. As special cases we obtain inclusion theorems for pairs of weighted mean matrices.

In [**5**], Sarigöl obtained necessary and sufficient conditions for $|N, p_n|_k \Rightarrow |N, q_n|_s$ for the case $1 \le k \le s$.

The concept of absolute summability of order k was defined by Flett [**3**] as follows. Let $\sum a_n$ be a given infinite series with partial sums s_n, and let σ_n^α denote the nth Cesaro means of order α, $\alpha > -1$, of the sequence $\{s_n\}$. The series $\sum a_n$ is said to be summable $|C, \alpha|_k$, $k \ge 1$, $\alpha > -1$, if

$$\sum_{n=1}^{\infty} n^{k-1} |\Delta \sigma_{n-1}^\alpha|^k < \infty, \tag{1}$$

where, for any sequence $\{b_n\}$, $\Delta b_n = b_n - b_{n+1}$.

In defining absolute summability of order k for weighted mean methods, Bor [**1**] and others used the definition

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{k-1} |\Delta u_{n-1}|^k < \infty, \tag{2}$$

where

$$u_n := \sum_{\nu=0}^{n} p_\nu s_\nu.$$

In using (2) as the definition, it was apparently assumed that the n in (1) represented the reciprocal of the nth main diagonal term of $(C, 1)$.